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MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Experimental Investigations on the Effective Temperature of the Sun, Made at Daramona, Streete, Co. Westmeath

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IX. *Experimental Investigations on the Effective Temperature of the Sun, made at Daramona, Streete, Co. Westmeath.*

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The expression "effective temperature of the sun" has by this time obtained a well-defined meaning, and may be taken (as stated by Violle and other physicists) to be that uniform temperature which the sun would have to possess if it had an emissive power equal to unity, at the same time giving out the same amount of radiant energy as at present.

The older estimates of this quantity were little more than guesses, and varied between 1500° C. and 3 to 5,000,000° C., or more.

The former of these values was given by assuming that Dulong and Petit's formula

$$R = ma^t,$$

where R = intensity of radiation, t = the temperature of the radiating surface, and m and a are constants for any one substance, held up to any limit.

The result given by it is obviously too low, as it is less than even the melting-point of platinum, the vapour of which probably exists in the solar atmosphere, and considerably lower than the temperature which may be obtained in the focus of a large lens.

The higher values were found by using Newton's law, in which radiation is taken as simply proportional to difference of temperature between the radiating body and its surroundings, a law which is proved to hold good only for very small differences.

It would appear, then, that by far the greatest difficulty in estimating the value of the solar temperature arose from ignorance of the law which connects the radiation from a hot body with its temperature, although there are minor difficulties which may still produce uncertainties in the final result.

One thing seems certain, that the temperature of the sun is far higher than any we can produce in our laboratories. This being so, the best that can be done is to make direct determinations of the connection between radiation and temperature within the widest possible limits, find an empirical law to which the observations conform, and trust that no break of continuity may make an extra-polation entirely useless.

So far, the only investigations made in this way appear to be those of LE CHATELIER* and ROSETTI.† LE CHATELIER measured the photometric intensity of the red light from solid bodies heated to different known temperatures, and obtained an empirical law which very fairly expressed his results from 700° to 1800° C.

He then, by passing sunlight through the same piece of red glass, measured the visual intensity of the "red radiation" coming from the sun, and, by applying the law just mentioned, deduced an effective solar temperature of 7600° C., which he admits to be an approximation with a possible error either way of 1000°.

The law he found is expressed thus :

$$I = 10^{6 \cdot 7} T^{-3210/T},‡$$

where I is the photometric intensity, and T the absolute temperature of the radiating body. On plotting the numbers that LE CHATELIER gives for corresponding values of I and T, it will be seen more easily than by mere inspection of the formula that I increases in an enormously rapid ratio as compared with T, which must evidently tend to vitiate the accuracy of the results obtained by extra-polation.

Then, as VIOLLE § points out, it is probable that the absorption by the red glass decreases as the radiation increases. And in discussing a question in which *total energy* as measured by heat is concerned, it is probably better to deal by experiment with the total energy than with a selected wave-length, or a group of wave-lengths.

Still the value thus obtained is sufficiently near those given by the utterly distinct methods of ROSETTI and of ourselves to increase considerably the probability of the approximate accuracy of our results.

ROSETTI attacked the problem in the most direct and complete manner hitherto attempted. He determined a law of radiation which held well up to 2000° C., and found in arbitrary units the heat radiated from an incandescent body at a known high temperature by means of a thermopile and galvanometer. He then measured the heat coming from the sun in the same units, and applied his formula to find the solar temperature, which finally came out at about 10,000° C. The questions of atmospheric absorption and the emissive powers of his incandescent solids were also investigated, and his work will be referred to more than once in the following pages.

* LE CHATELIER, 'Compt. Rend.,' 1892, vol. 114, p. 737.

† ROSETTI, 'Phil. Mag.,' 1879, vol. 8, 5th series, pp. 324, 438, 537.

‡ The negative sign in the exponent is omitted in LE CHATELIER'S paper, probably by a mere slip.

§ VIOLLE, 'Compt. Rend.,' 1892, vol. 114, p. 734.

I. General Method and Instruments.

The general idea in this investigation was to endeavour to *balance* the heat of the sun by means of an artificial source of heat at a high known temperature, thus obtaining both directness and simplicity as far as possible. The artificial source of heat was a strip of platinum heated by an electric current; this strip formed part of a modified form of JOLY'S Meldometer, which is described below, and its temperature could be determined at any moment with a high order of accuracy.

The radiation from a known area of the incandescent strip was balanced against that coming from the sun in a differential radio-micrometer—a modified form of Professor BOYS'S well-known and excessively delicate instrument.

The essential theory of the method was extremely simple. Knowing the apparent areas of the sun and the artificial source of heat (the latter, of course, being much the greater), and knowing the law connecting radiation and temperature, we can at once find to what point the latter would have to be raised to balance the sun, if these apparent areas were made equal. But this would be the required effective temperature of the sun, if the emissive powers were equal, and both bodies could radiate directly and without intervening absorption on to the receiving surface of the radio-micrometer.

This extreme simplicity, however, cannot be obtained, and correcting factors have to be applied for—

(*a*) Emissive power of the platinum strip;

(*b*) Reflecting power of the glass in the heliostat, which keeps the beam of sunshine in the required position;

(*c*) Terrestrial atmospheric absorption.

Each of these will be discussed in turn, after the instruments used have been described.

The general arrangement of the apparatus is shown in fig. 1.

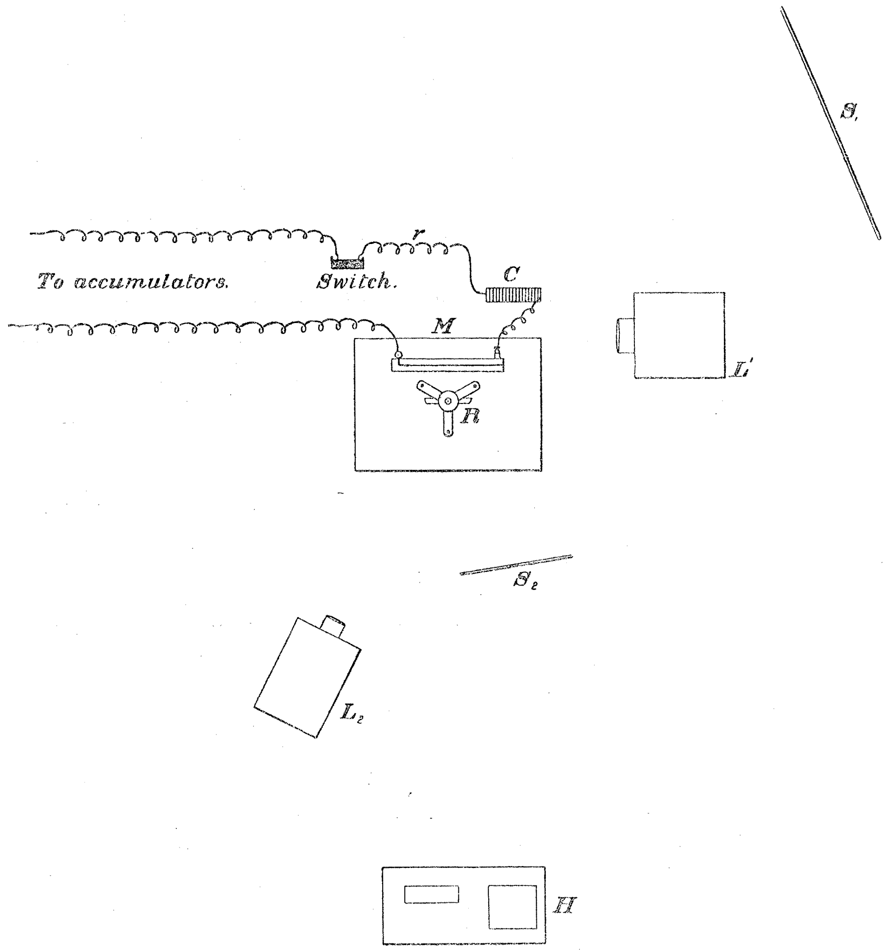
H is the heliostat, which is placed on a window sill outside the laboratory, about 4 metres from the radio-micrometer R, and the meldometer M. The two latter instruments are supported on a table which stands on a concrete pier passing through the floor of the room.

S_1 is the scale of the meldometer, the distance from S_1 to M being about 3 metres. S_2 is the scale of the radio-micrometer, and L_1 and L_2 are the lamps corresponding to the two instruments. C is a variable carbon resistance; r is a platinoid coil; C_1 and the platinum strip in M are in circuit with 26 EPSTEIN accumulator cells, by means of which the strip is heated to any desired temperature.

In an experiment, a beam of sunlight is reflected on to the receiving surface of one circuit—say, the lower—of the radio-micrometer, and the heat from the platinum strip on to that of the higher; the two circuits are arranged so that, under these conditions, the two sources of heat produce turning moments in opposite senses, and the temperature of the platinum is raised until a balance is obtained, indicated by the index spot of light returning to its zero on the scale of the radio-micrometer.

At this same moment the temperature-scale of the meldometer is read, the local time of the observation is noted (to obtain the altitude of the sun), and a reading on the heliostat is made, by which the angle of incidence of the sunlight on the mirror can be calculated.

Fig. 1.



An exactly similar process is then gone through with the sun shining in the upper circuit and the platinum in the lower, and the results of each observation are separately calculated.

Then if $R_p =$ the radiation in our arbitrary units, corresponding to a balancing temperature,

$A =$ the ratio of the total heat to the amount transmitted at the observed altitude of the sun,

$b =$ the ratio of the incident radiation to that reflected from the mirror of the heliostat,

$c =$ the ratio of the apparent areas of the platinum and the sun,

and $d =$ the ratio of the emissivity of bright platinum compared with that of lamp-black,

then R_s, the radiation from the sun outside our atmosphere, will be

$$R_s = R_p \times c \times A \times b \times d.$$

The Meldometer.

The meldometer in its original form was devised by Professor JOLY,* for the purpose of finding the melting-points of minerals, hence its name.† As used by him, it consists of a strip of platinum, on which minute fragments of any mineral can be placed, while any alteration in its length can be determined by means of a micrometer screw which touches a lever connected with one end of the strip.

The strip can be heated by an electric current, and is calibrated by observing the micrometer readings corresponding to the temperatures at which some substances of known melting-points melt.

The first alteration which we made on the original form of instrument was to substitute an optical for a mechanical indication of the expansion of the strip, by means of which an alteration in length, due to a rise of 1° C. in temperature, could be detected.

For purposes of calibration it is convenient to place the plane of the strip horizontal, so that the fragment of selected material may rest upon it, and this was the arrangement in our first instrument.

But this introduces the necessity of a mirror at 45° to reflect the heat from the strip into the radio-micrometer—a serious source of error, as no good series of experiments on the reflecting power of speculum metal is to be found, and even if it were, tarnishing of the surface is bound to take place, and make the reflection irregular.

We had, therefore, to solve the problem of keeping our thin strip in a vertical plane, while at the same time supporting fragments of our selected minerals upon it during the calibration experiments. The plan finally adopted was to turn up a very narrow ledge along one edge of the strip, at right angles to the remainder, this ledge serving with very careful handling, as a support for the mineral fragments. A cross section of the strip was thus L-shaped, but with a very short horizontal arm, thus:

* 'Proc. R. Irish Acad.,' vol. 2, 3rd series, 1891, p. 38.

† We have thought it better to retain Professor JOLY'S name, although it no longer describes the function of the instrument as used in our work.

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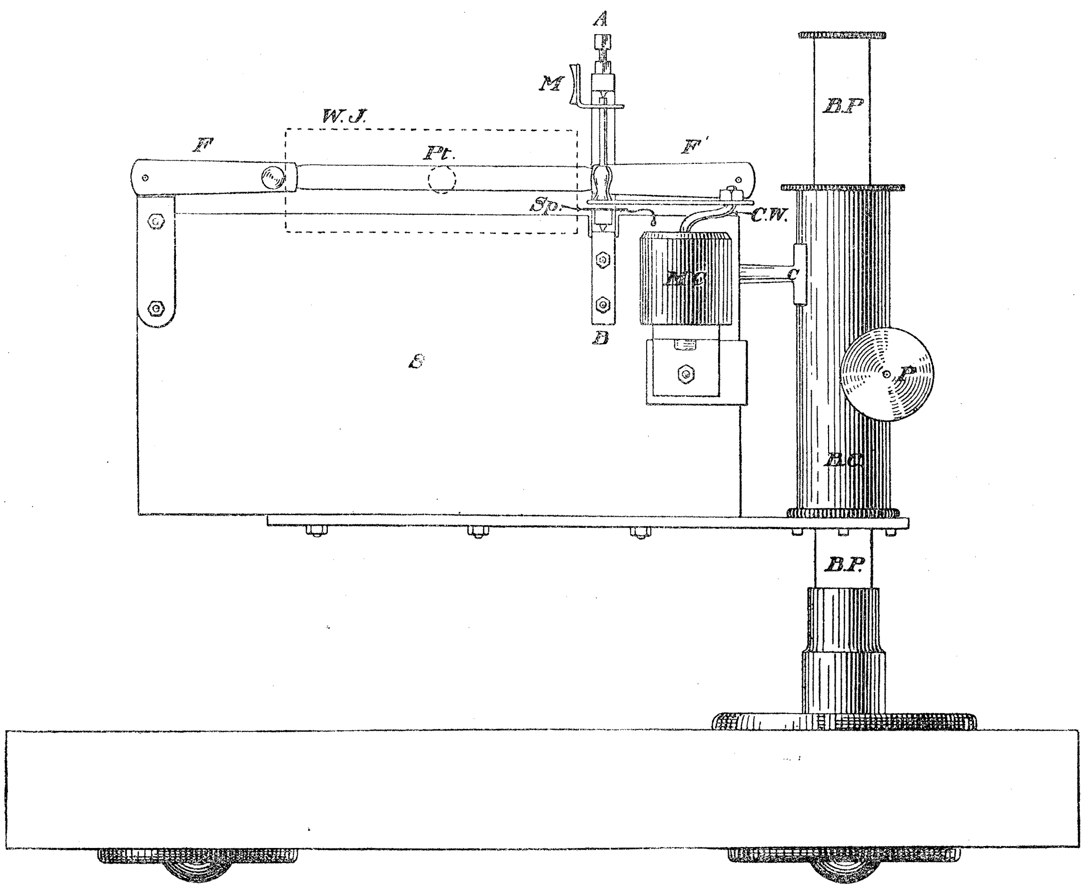
The dimensions of the strip were :—

Length	102	millims.
Breadth (including ledge)	12	,,
Thickness	0·01	millim.

Fig. 2 shows the final form of the instrument with the water-jacket removed. It was made by Messrs. YEATES and SONS, Dublin.

Fig. 2.



The Meldometer. Scale, about $\frac{1}{2}$.

S is a block of slate, $17\frac{1}{2} \times 9 \times 3$ centims., rigidly fastened to a cylinder of brass, *B.C.*, which can be worked up and down a square brass pillar, *B.P.*, by means of the pinion *P*.

The pillar is screwed firmly to a heavy slate base-plate, on which the radio-micrometer also stands. The platinum strip, *Pt.*, is held between two forceps, of which one, *F*, is fixed, and the other, *F′*, is free to rotate on an axle which is supported between *A* and *B*. In this way the jaws of the forceps, *F′*, which hold the strip between them, can move, when the strip expands, in a small circular arc, which

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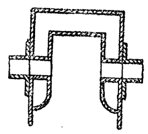
in the experiment is not far from a straight line. *M* is a concave mirror fixed to the axis of rotation; it gives the image of a luminous slit on a straight scale, 3 metres away, and thus indicates an expansion of the strip, as already explained. A piece of stout copper wire, *C. W.*, is connected with the forceps, and dips into a mercury cup, *M.C.*, by means of which a movable electric connexion is maintained with the remainder of the circuit. *Sp.* is a flat spiral spring, which is necessary to keep a slight tension on the strip. A water-jacket of gilded brass (shown in dotted lines) rests on the top of the slate block during an experiment; its shape is shown in fig. 2A, which is a cross-section; its length is a little greater than that of the strip,

Fig. 2A.



Section of Water-jacket.

and in the middle of each of its long sides is a circular hole through either of which the heat of the incandescent platinum passes, the hole not in use being plugged up with a gilt brass cap. The water-jacket serves two purposes: one is that of protecting the glowing platinum from air currents, which would otherwise tend to produce quick variations in its temperature; the other is that of preventing any radiation from the platinum except that which passes through the aperture into the radio-micrometer.

Calibration of the Platinum Strip.

The platinum was obtained from Messrs. JOHNSON, MATTHEY, and Co., Hatton Garden, London, who reduced it in thickness until a convenient current (25 ampères) from the accumulators was able to raise it to full incandescence.

The calibration experiments were performed as follows:—

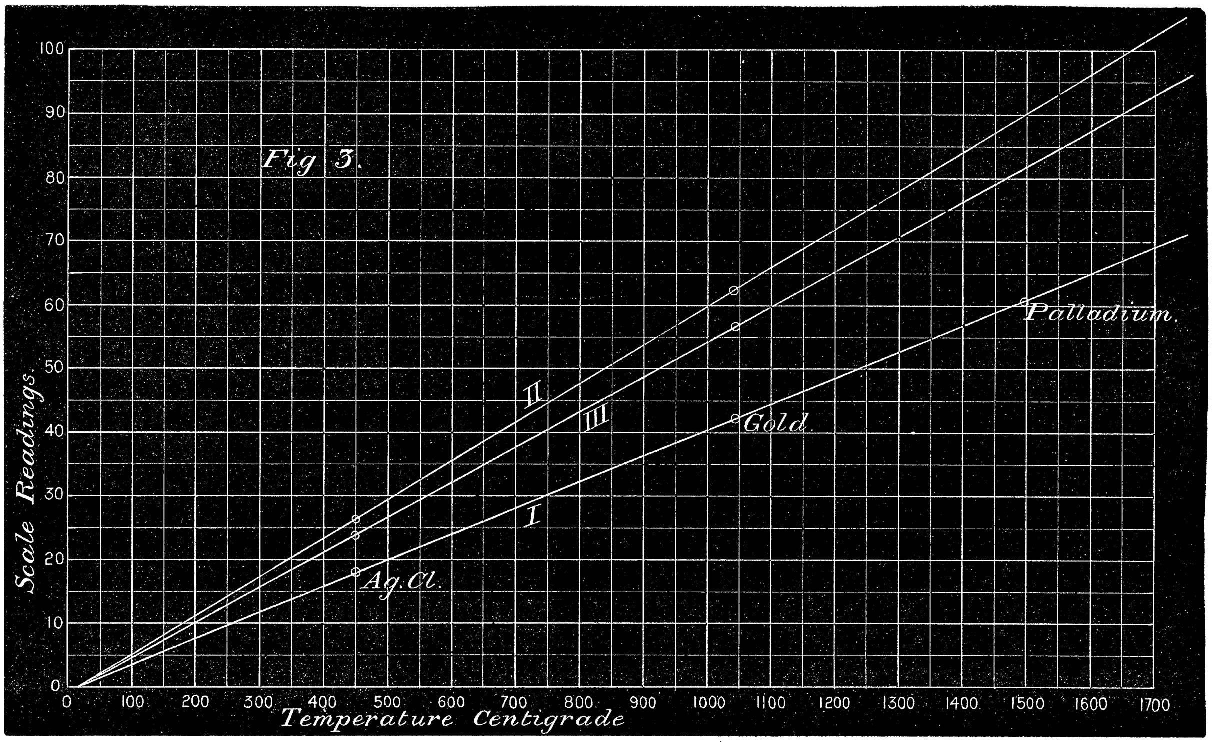
The mirror connected with the strip was turned until the reflected spot of light occupied a convenient position on the scale, which stood at a distance of about 3 metres, and was placed at right angles to the zero position of the index beam of light. A very small fragment of silver chloride (approximately $\frac{1}{12}$ of a milligramme in weight) was then placed on the platinum strip, near the middle of its length, and a low-power microscope was so held in a clamp that the fragment could be plainly seen through an aperture in the water-jacket. The melting point of AgCl is taken as 451° C. (on the authority of CARNELLEY*), at which point the platinum was under a red heat, so that a candle had to be arranged to shine through an open end of the water-jacket, the gilt sides of which reflected the light so well on to the silver

* CARNELLEY "Melting and Boiling-points Tables."

chloride that it stood out with great distinctness against the dark metal in the field of the microscope.

One observer, with his eye at the microscope, then switched on the current, and very slowly raised the temperature of the strip by turning the compressing screw of the carbon-resistance, until a sudden definite melting of the fragment took place ; at the same moment the second observer took the reading on the scale, which reading then indicates the temperature 451° C.

Fig. 3.



An exactly similar process was gone through, using a minute piece of chemically-pure gold (in weight about $\frac{1}{6}$ of a milligramme), the melting-point of which we took as 1041° C. A curve was then drawn in which the abscissæ are temperatures and the ordinates scale readings. One point on the curve is evidently 0 on the scale at 15° C. (the temperature of the room). The other two points, viz., those corresponding to melting gold and melting AgCl, *lie exactly on a straight line* with this first point. That this coincidence was not mere chance is proved by the fact that we have calibrated three different strips—one in the first meldometer, in which the plane of the strip was horizontal, and two in the second instrument, with the plane of the strip vertical. The straightness of the line in each case is as perfect as it can be drawn with a straight edge.

The figures for the three strips are :

	Melting substance.	Temperature.	Deflection from zero.
		° C.	
1st strip	Ag.Cl.	451	18·1 }
1st strip	Gold	1041	42·0 }
2nd strip	Ag.Cl.	451	26·4 }
2nd strip	Gold	1041	62·1 }
3rd strip	Ag.Cl.	451	24·2 }
3rd strip	Gold	1041	56·8 }

NOTE.—VIOLLE gives the melting-point of gold as 1045° C. CALLENDAR, 'Phil. Mag.,' vol. 33, 1892, gives 1037° C. The mean, 1041° C., of these modern determinations cannot be far from the truth.

The three lines thus given are shown in fig. 3.

In the case of the 1st strip, a piece of palladium was also tried, the melting-point of which is given by VIOLLE as 1500° C.; a deflection of 61 was obtained on the scale, which falls exactly on the line given by the other two substances.

By means of the straight line, corresponding to the particular strip of platinum, therefore, the temperature of the latter may be known with a high degree of accuracy by reading the position of the spot of light on the thermometer scale, on which 1 millim. corresponds to about 2° C.

JOLY,* in his paper, refers to the possibility of a viscous extension of the platinum after being raised to high temperatures; we have proved that this does not take place in our experiments, by noticing that the spot of light returns exactly to zero very soon after the current is cut off, when the platinum has been for some 15 seconds at a temperature of over 1500° C.

The Differential Radio-micrometer.

This instrument is a modification of the single form described by Professor BOYS.† The chief difference consists in a duplication of the circuits, both circuits being supported by the same fibre. The remaining changes consist in an alteration of the position of the magnets, &c., which for our purpose are more conveniently placed vertically instead of horizontally. It was constructed by Messrs. YEATES and SONS, Dublin, and the double circuit by Mr. W. WATSON, B.Sc., of the Royal College of Science, London.

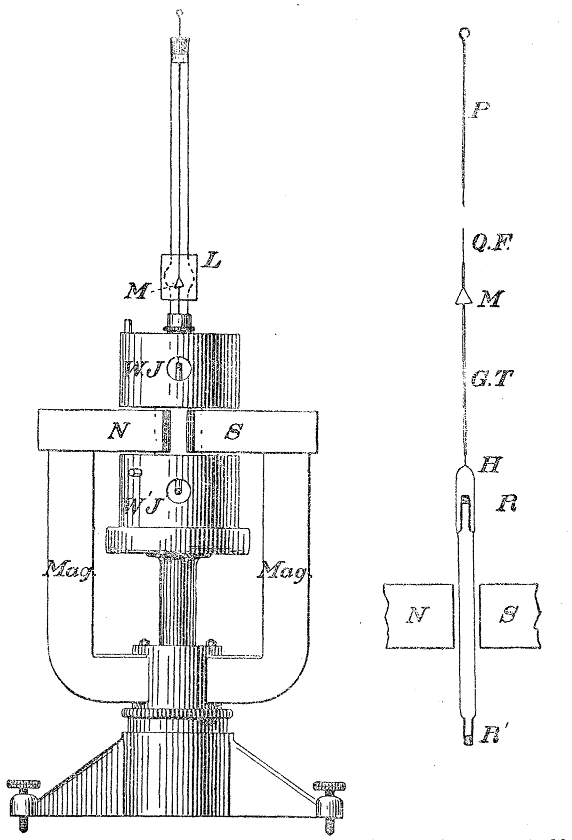
The instrument is shown in elevation in fig. 4, on a scale of about $\frac{1}{4}$, while the circuit is shown *about* $\frac{1}{2}$ *size* on the right of the figure, where R, R' are the two receiving surfaces of blackened copper foil, attached to which are the bars of the alloys. The two pairs of bars are connected by a circuit of fine copper wire, and the whole system is supported by a hoop (H) of similar wire (from which, of course, it is

* JOLY, 'Proc. Roy. Irish Acad.,' 1891, 3rd series, vol. 2, p. 61.

† C. V. BOYS, 'Phil. Trans.,' vol. 180, 1889, A., p. 159.

insulated) to a fine glass tube, *G.T.*, to which is fastened the mirror, *M.* The quartz-fibre suspension, *Q.F.*, is held by the pin, *P*, which passes through a cork, as shown in

Fig. 4.



Scale, about one-fourth. Scale, about one-half.

The Differential Radio-micrometer and Circuit.

the quarter-scale drawing. The weight of the entire system below the pin is about $1\frac{1}{2}$ grain.

In the elevation of the complete instrument, *Mag.* denotes the magnet, *N* and *S* the pole pieces, between which the circuit hangs inside a hollow block of brass, with an iron core as in the ordinary form of the radio-micrometer. *L* is a lens, which, with the small mirror, *M*, forms an image of a luminous slit, on a scale at a distance of about a metre.

W.J. and *W'.J'.* are water-jackets, through which it was found better not to allow the water to circulate. They were kept filled, however, to prevent sudden changes of temperature from affecting the circuits.

The lower water-jacket rests upon a disc of mahogany, which is supported by a brass pillar; the details of the remaining parts of the instrument will be obvious on an inspection of the diagram.

The water-jackets are pierced by tubes, through which the receiving surfaces are

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visible, and by means of which heat can be allowed to fall upon them. If desired, any or all of the tubes may be stopped by means of corks.

In an experiment, a short tube is inserted in the opening in the water-jacket opposite to the receiving surface, on which the heat from the platinum is to be allowed to fall; the mouth of the tube is partially closed by a stop of polished brass, in which is a circular hole, 4·94 millims. in diameter; the size of the aperture was carefully measured by means of a micrometer gauge. The distance of the aperture from the receiving surface was also carefully measured, and is equal to 60·2 millims.

This gives for the angle subtended by a diameter of the aperture at the receiving surface, $4^{\circ}\cdot 702$.*

This number is a constant for any position of the strip, and is equal to the apparent diameter of the disc of glowing platinum as seen from the receiving surface; the distance of the platinum strip, therefore, may be altered without affecting the reading of the radio-micrometer, provided that it be not so great that the angle subtended by its width is less than that subtended by the aperture. In the hole in front of the receiving surface, on which the heat of the sun falls, a brass tube, 8 centims. long, and blackened inside, is inserted to cut off side radiation. A wooden box covers the entire instrument during an experiment, the box containing holes opposite to those in the water-jackets. By this means the instrument is completely protected both from draughts and from accidental radiation from lamps or other sources of heat in the room.

Fig. 5 is from a photograph, showing the radio-micrometer and meldometer in position, with the protecting wooden cover of the former removed.

The Heliostat.

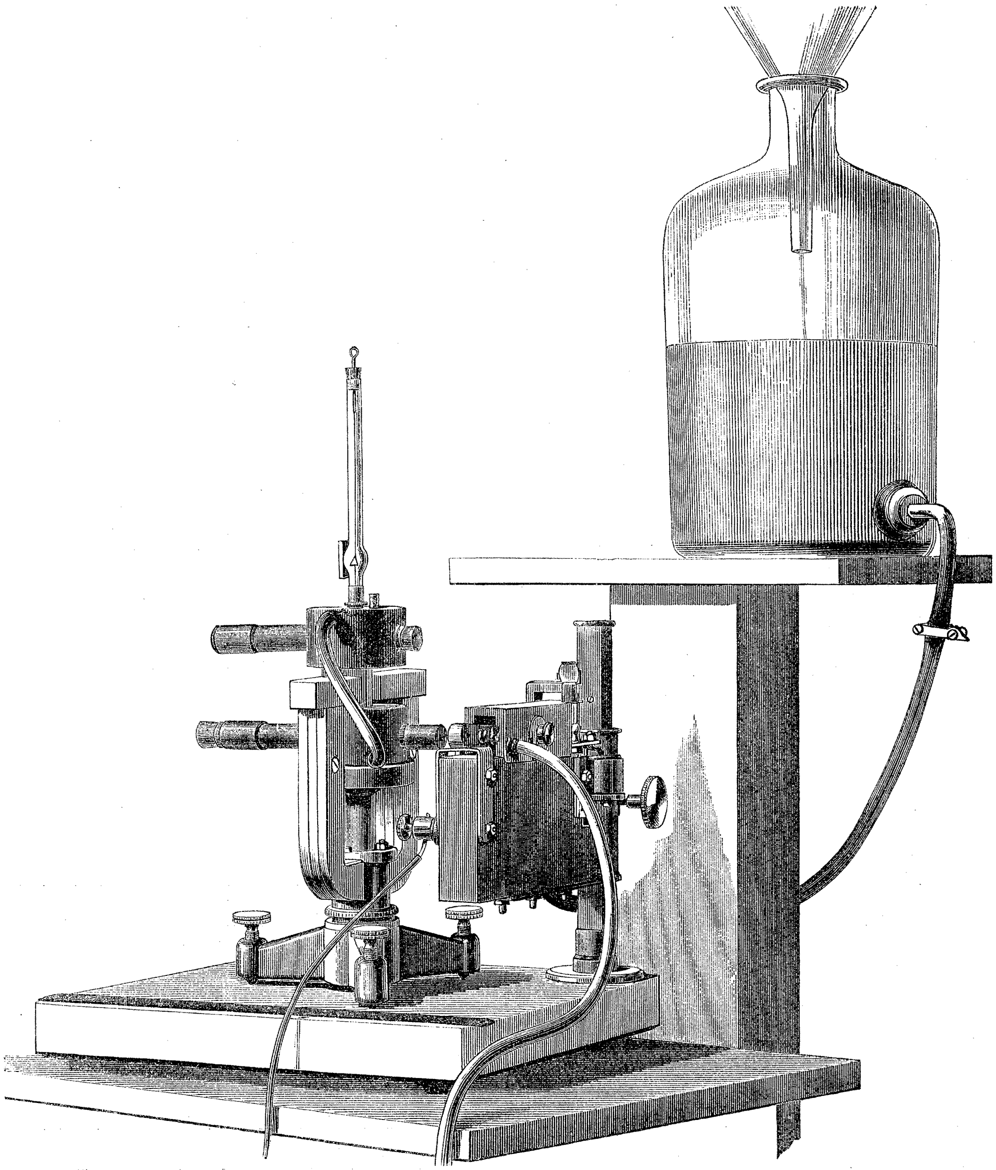
The heliostat used was a single-mirror instrument of Professor G. JOHNSTONE STONEY's design. The mirror was a thick piece of plate glass, with a plane surface carefully figured by Sir HOWARD GRUBB. It was unsilvered, and well blacked at the back, and was of such dimensions that it subtended an angle at the radio-micrometer, when inclined at its usual angle during our experiments, only a little larger than that subtended by the sun. The sunlight from the mirror passed through a small hole in the shutter of the laboratory window, and by this arrangement the heat from the sky round the sun was completely cut off; thus no measurements had to be made, as in Professor ROSETTI's work, to obtain the effect of sky radiation.

The use of a single-mirror heliostat was essential, on account of the irregularities produced by polarization in the intensity of the beam reflected from two surfaces, as well as from the difficulty of measuring the two angles of incidence in a two-mirror form.

* See note on p. 391.

The question may arise as to whether it is correct to consider the reflection from the front surface of the heliostat mirror only, or whether multiple reflections from the back surface might not appreciably increase the total amount of heat reaching the radio-micrometer. That the former idea is correct will be evident from the following considerations :—

Fig. 5.



The glass of the mirror was sufficiently thick to clearly separate (at the angles of incidence ordinarily used in our experiments), the image given by the first ordinary reflection from the first given after a "back-reflection," supposing such to exist. We focussed a telescope on the image of the sun in the mirror, but could not discover even a faint ghost of a second image, thus showing that, at least for all wave-lengths in the visible spectrum, there was no regular reflection from the back surface. Even if the black varnish happened to possess a refractive index equal to that of the glass, the virtual effect would merely be a slight thickening of the plate, and it would still hold that all the energy due to what we may call for brevity, the "visible wave-lengths," reaching the back surface, was there absorbed and then diffused in every direction, the amount reaching the radio-micrometer on this account being absolutely negligible.

As for the ultra-red vibrations, it would be unreasonable to suppose that when all the "visible wave-lengths" were absorbed, there should be a rapid change in the nature of the back-reflections, so that a "dark image" might be reflected when no sign of a "light image" was to be found. Moreover, if such a condition could be considered likely, the additional radiation must be extremely small, as we know that by far the greater portion of the heat-energy of the solar radiation is contained within the limits of the visible spectrum.

The point hardly needed further confirmation, but as a check on the curve (fig. 9), obtained from FRESNEL'S formula, we made three photometric observations, as mentioned elsewhere (p. 386), which gave points very nearly on the theoretical curve.

ON THE LAW CONNECTING RADIATION AND TEMPERATURE.

We have already mentioned some experiments which have been made in this part of the subject, and seen that it is ignorance of the law which has been the main cause of disagreement in the final estimation of the solar temperature.

ROSETTI'S experiments on this point were divided into two parts. He first found the effect on his thermopile of the radiation from a cube filled with water, and afterwards with mercury, at temperatures from about 60° to 300° C. He then found an empirical formula which closely expressed the observed results. The law is expressed thus—

$$y = a\mathrm{T}^2(\mathrm{T} - \theta) - b(\mathrm{T} - \theta),$$

where

y = the thermal effect of the radiation as given by the deflections on the scale of the thermopile,

T = the absolute temperature of the radiating body,

θ = the absolute temperature of the medium surrounding the body on which the radiation falls;

while

a and b are constants which must be determined from two corresponding values of y and T.

Experiments were then made with the radiating body at higher temperatures, which were obtained either by holding a disc of metal in the flame of a Bunsen burner, or by heating oxychloride of magnesium in the oxyhydrogen flame, preliminary experiments having been made on the emissive power of the various substances at these high temperatures.

Some little doubt must necessarily exist as to the power of knowing exactly what these temperatures actually were; nevertheless, the results obtained appear consistent and trustworthy, and the accuracy of the parabolic formula was tested satisfactorily up to a temperature of something like 2,000° C.

In our experiments, the heat from the platinum strip was, with our first meldometer, allowed to fall on a mirror of speculum metal at 45°, and thence into the radio-micrometer. The temperature of the platinum was raised step by step, and, at each step, the deflections, both of the temperature scale and of the radio-micrometer, were noted.

Numerous sets of experiments were made, but with some want of uniformity in the results. At first it appeared that STEFAN'S* law of the fourth power expressed the results; then, with additional precautions, ROSETTI'S law appeared to be confirmed. But the want of knowledge as to the reflective power of the speculum metal, with the alterations in the state of its surface, as well as difficulties in throwing the reflection of the glowing platinum fairly into the radio-micrometer, prevented our acceptance of any of these results as beyond suspicion.

With the second meldometer, the need of a mirror was obviated; the differential radio-micrometer was replaced by one of the ordinary single form, perfectly protected against accidental radiations, and, finally, three independent series of experiments gave concordant results which may be very closely expressed by a *fourth power law.*

The radiation is taken as proportional to the deflections on the scale of the radio-micrometer, which was at a distance of about 123 centims.; the extreme angular deflection was about 20°, and up to these limits the proportionality is proved to hold accurately.†

The curve (fig. 6) is calculated from the formula

$$R = a\,(T^4 - T_0^4),$$

where

R = the radiation expressed in scale-readings,

T = the absolute temperature of the incandescent platinum,

T_0 = the absolute temperature of the medium surrounding the radio-micrometer (*i.e.*, temperature of the room),

and

a is a constant which was calculated from four points on the experimental curve.

In this case, $\log a = \overline{11}\cdot67868$.

The temperature of the room being about 15° C. = 288° absolute, then R = 0, $T = T_0 = 288°$, will give a point both on the experimental and the calculated curves.

* STEFAN, 'Wien. Ber.,' vol. 79, (1), 1879, p. 391.

† See p. 378.

It will be noticed at once that at comparatively low temperatures the curve does not accurately express the facts, but that the agreement is very good as the temperature rises. This disagreement has been confirmed by LECONTE STEVENS, whose paper* came under our notice after our experiments were finished and the curve drawn. He concludes that, at comparatively low temperatures, the fourth power law gives too rapid a rate of increase of radiation, which agrees with our observations, but that as the temperature rises this divergence diminishes.

The following table gives the results of the three series of observations, which are also plotted on the curve, fig. 6; in two cases, the difference between the observed and calculated results is so large that some misreading seems likely, otherwise the agreement is very satisfactory :—

TABLE I.

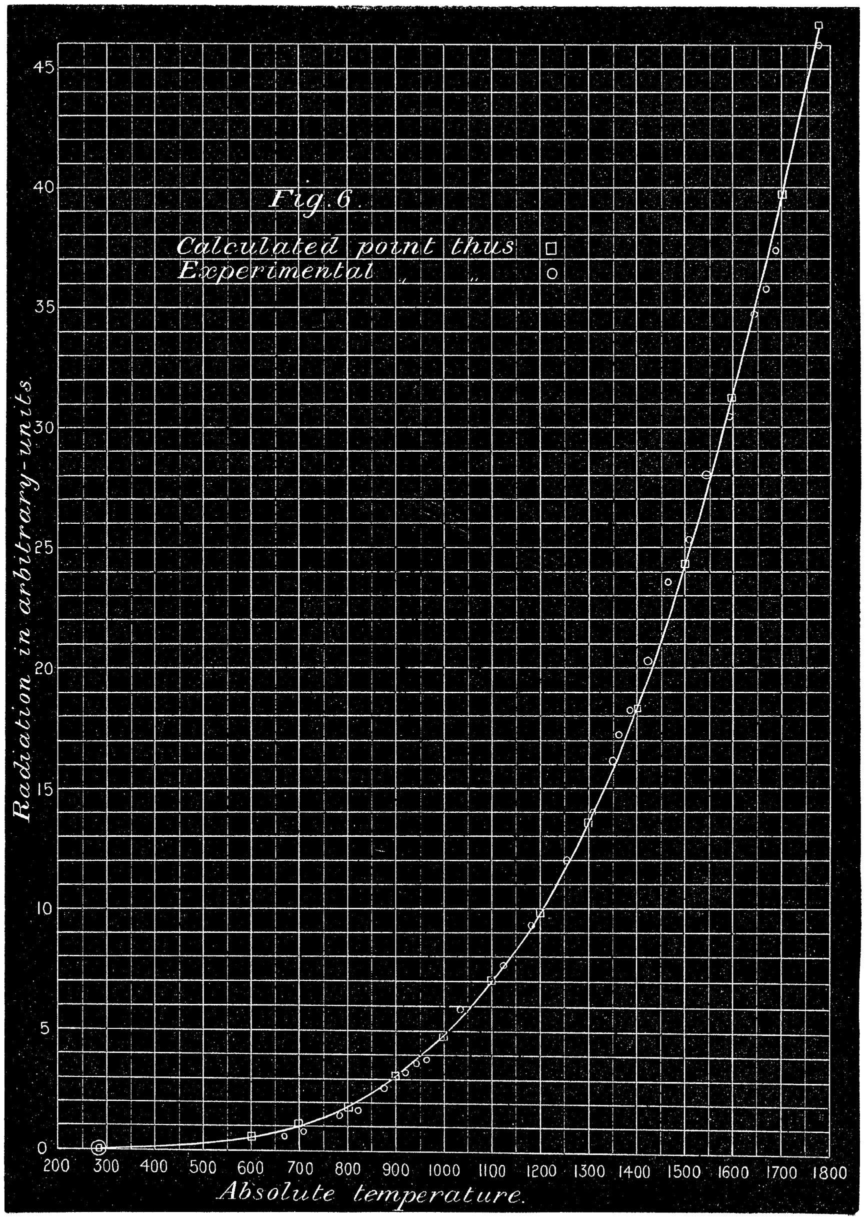
Temperature absolute.	Radiation.		Calculated—observed.
	Observed.	Calculated.	
° 288	0	0	0
671	7	9	+ 2
703	9	11	+ 2
788	16	18	+ 2
811	18	20	+ 2
876	26	27	+ 1
915	32	33	+ 1
944	37	37	0
965	39	41	+ 2
1045	59	57	− 2
1125	76	76	0
1181	93	93	0
1253	120	119	− 1
1308	140	140	0
1348	161	158	− 3
1363	172	159	(−13)
1393	182	180	− 2
1425	202	198	− 4
1466	236	220	(−16)
1513	253	252	− 1
1547	280	272	− 8
1593	305	306	+ 1
1647	348	348	0
1663	358	360	+ 2
1683	373	380	+ 7
1773	460	462	+ 2
		Mean. . .	$\frac{+24-50}{26} = -\frac{26}{26} = -1$

* 'Amer. Jour. of Science,' vol. 44, 1892, p. 431.

Or, omitting two obviously bad observations, the mean difference between "calculated" and "observed"

$$= \frac{+ 24 - 21}{26} = \frac{+ 3}{26} = + 0 \cdot 1.$$

Fig. 6.



The latest work on this subject is that of PASCHEN,* who gives full references to the papers of other experimentalists. His method of working is very complicated,

* 'WIEDEMANN'S Annalen,' vol. 49, 1893, p. 50.

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and the determination of his high temperature appears to be wanting in certainty. He finally obtains results which do not agree with any formula hitherto given.

The least disagreement is found with an empirical expression given by WEBER,* but PASCHEN'S curve (in which, as in our own, the abscissæ are temperatures, and the ordinates radiation) falls nearly as much below WEBER'S as it rises above STEFAN'S. Taking, as a particular instance, PASCHEN'S observed radiation at 1273° and 1673° (absolute) = 69 and 295 approximately, the fourth power law gives 50 and 148, while WEBER'S gives 76 and 570.

PASCHEN'S results would therefore indicate a much more rapid rise in radiation than that indicated by our fourth power law; in the case just quoted the exponent would be about 5·3.

We are supported, however, in our adoption of the fourth power law, not only by our own and STEFAN'S results, and LECONTE STEVENS' conclusions, but also by some work of SCHNEEBELI,† and in a very interesting way by an investigation of BOLTZMANN'S,‡ who deduces the law from the electro-magnetic theory of light.§

On the whole, therefore, we think there can be little doubt that, at least in the case of incandescent platinum, the increase of radiation with temperature may be most accurately expressed by the fourth power law, and that the divergent results obtained by different investigators are chiefly due to want of certainty in the determination of high temperatures, and in a less degree to complication of apparatus, with its accompanying accumulation of small errors. In the case of our own experiments, the temperature of the platinum strip is known with a doubt of only some 6° C. at a temperature of 1500° C.; the radiation falls directly on the radio-micrometer, and the proportionality of the deflections of the latter to the radiation falling upon it is strictly demonstrated by experiment. It would seem, therefore, that the results cannot be far from the truth, which conclusion is largely strengthened by the confirmations already mentioned.

It has been generally assumed that the deflections of the spot of light on the scale of the radio-micrometer are proportional to the amounts of radiation falling on the receiving surface of the instrument. In the above experiments the extreme deflection was about 20°, and it therefore seemed necessary to determine by direct experiment whether this proportionality held up to this high limit or not. This was done in the following manner:—

A cube of boiling water was supported at a distance of about 80 centims. from the

* H. F. WEBER, 'Berlin Akad. Ber.,' 1888, 2, p. 933.

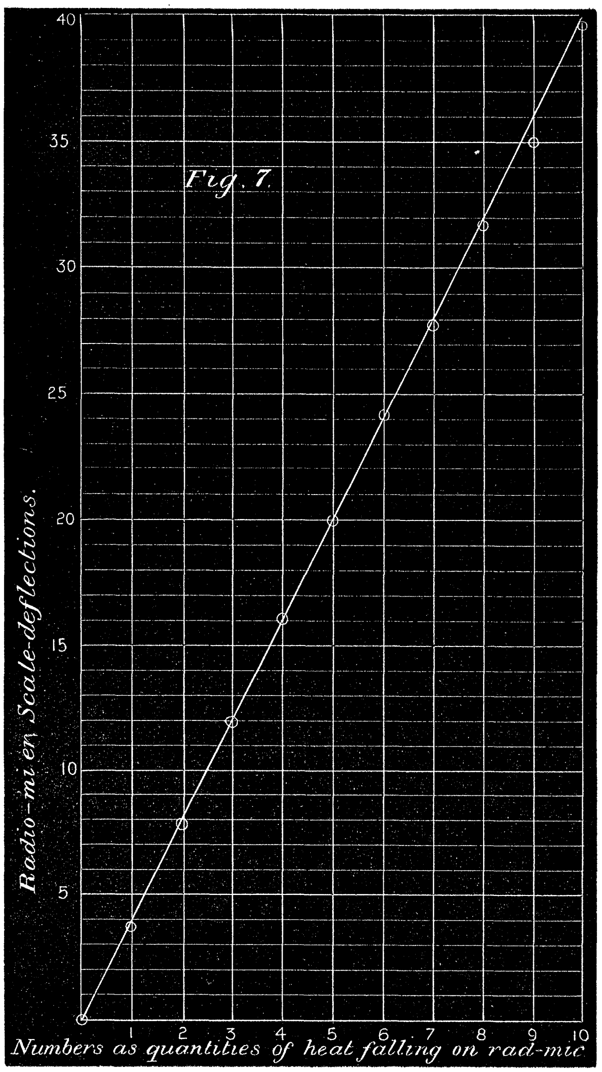
† SCHNEEBELI, 'WIEDEMANN'S Annalen,' 1884, vol. 32, p. 403.

‡ BOLTZMANN, 'WIEDEMANN'S Annalen,' 1884, vol. 32, pp. 31 and 291.

§ [It must be noticed, however, that both STEFAN'S and BOLTZMANN'S results were supposed to apply, strictly speaking, to "pure" radiation from a surface of unit-emissive power, so that the agreement must not be insisted on too strongly. All we can say certainly is that, for the particular results of particular experiments, the fourth power law is found to hold very accurately, and has therefore been adopted.]

radio-micrometer; between the two a wooden box, 4 inches square in section, was placed to prevent side radiation from disturbing the latter; tin and cardboard screens were also used for the same purpose, until we were assured that the only heat falling on the instrument was that from the lamp-blacked side of the cube, passing through a carefully-cut rectangular aperture, made in cardboard and fixed to the end of the

Fig. 7.



wooden box close to the cube. A horizontal edge of the aperture was divided into ten equal parts, and a wooden screen, with a straight edge, could be placed so as to close the aperture, or to leave any desired fraction of it open. The proportionate area of aperture open, and therefore the proportionate amount of heat falling on the instrument, was then given by the reading of the scale on the horizontal edge of the aperture.

The following Table II. gives the results of two series of experiments. The first

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column gives the area of aperture, *i.e.*, the quantity of heat falling on the instrument; the second gives the deflections (in centims.) on the scale, in the two series; the third gives the mean; and the fourth gives the deflections calculated by a straight line formula, $y = mx$.

When the observed results are plotted down on curve paper (fig. 7), it will be seen at once that they form as nearly as can be a straight line; and as the extreme deflection in these cases was $21\frac{1}{2}°$, the proportionality of radiation and deflection is strictly demonstrated, up to the greatest value of the latter used in our experiments.

TABLE II.

Quantity of heat.	Deflection.	Mean observed.	Calculated from $y = 3·96x$.	Observed − calculated.
0	0·0	0·0	0·0	0·0
1	4·4, 2·9	3·7	4·0	− 0·3
2	8·6, 7·2	7·9	7·9	0·0
3	12·5, 11·3	11·9	11·9	0·0
4	16·8, 15·4	16·1	15·8	+ 0·3
5	20·7, 19·6	19·9	19·8	+ 0·1
6	24·4, 23·9	24·2	23·8	+ 0·4
7	27·9, 27·9	27·9	27·7	+ 0·2
8	31·5, 31·8	31·7	31·7	0·0
9	34·9, 35·3	35·1	35·6	− 0·5
10	39·6, 39·6	39·6	39·6	0·0
				Mean $= \frac{+1·0 - 0·8}{11} = + ·02$.

It may be noticed here that as the temperature rises, ROSETTI's law becomes more nearly a simple third-power law, while ours becomes a simple fourth-power law, so that if

R_p = radiation from platinum,
T_p = temperature of platinum,
R_s = radiation from sun,
T_s = temperature of sun,

then

$$\frac{R_s}{R_p} = \frac{T_s^4}{T_p^4}, \quad \text{or} \quad T_s^4 = T_p^4 \times \frac{R_s}{R_p},$$

which gives when $T_s = 6000°$ and thereabouts, a result differing by less than one degree from that obtained by the complete formula $R_s = a(T_s^4 - T_o^4)$.

The simple form gives a great saving of time in calculating out the results of the observations, and we generally adopted it in the course of our work. The only direction in which we can look for an explanation of the great difference between ROSETTI'S law and our own, is in that of his method of estimating his high temperatures, which appear to be somewhat uncertain, whereas we can feel confident in the accuracy of our own method to within $\pm 6°$ at 1500° C. The chances are that his discs of metal were at a lower temperature than that assumed (but not measured) by him; and if that were so, the differences between his results and ours would be in the direction in which we find it.

THE EMISSIVE POWER OF PLATINUM AT HIGH TEMPERATURES.

SCHLEIERMACHER* and ROSETTI† have made experiments on this subject which at first sight appear to disagree, but on examination confirm one another in an interesting manner. From the curves which SCHLEIERMACHER'S results give, we obtain the emissions at certain temperatures (1) from polished platinum, (2) from platinum covered with black oxide of copper, which may be assumed as approximately the same as that from a lamp-black surface. The fourth column in the following table gives the ratio of the two emissions :—

Absolute temperature.	Emission.		Ratio $\frac{\text{black}}{\text{bright}}$.
	Plat. (black).	Plat. (bright).	
° 300	65	12	5·42
400	96	20	4·80
500	147	34	4·32
600	220	52	4·23
700	317	77	4·12
800	445	112	3·97

The figures in the fourth column show a gradual fall in the ratio as the temperature rises. ROSETTI, at an absolute temperature of about 1500°, found for the ratio $100/35 = 2·9$, which falls in fairly satisfactorily with a theoretical continuation of SCHLEIERMACHER'S results. As it is impossible, with our present arrangement of apparatus, to keep the platinum lamp blacked at a high temperature, and as the ratio is evidently altering very slowly near the point at which ROSETTI made his determinations, we shall use his ratio in calculating our results, *i.e.*, we shall take

$$\frac{\text{Emission from lamp black}}{\text{Emission from bright platinum}} = \frac{100}{35} = 2·9.$$

* 'WIED. Ann.,' 1885, vol. 26, p. 287.

† 'Phil. Mag.,' vol. 8, 1879, p. 445.

The Atmospheric Absorption.

Until LANGLEY* published his "Researches on Solar Heat," the unanimity with which nearly all observers agreed in giving a value of about 21 per cent. to the absorption of light and heat from a radiating body in the zenith, was so striking that there seemed little doubt as to the practical accuracy of this figure. Yet, in every case, since under most favourable conditions the experiments must have been done with a thickness of at least *one* atmosphere, an assumption had to be made as to the effect which would have been produced without this thickness, and Professor LANGLEY showed conclusively that this assumption was not justified by the conditions of the problem.

The formula which had been most generally accepted as expressing the amount of radiation received from a body at different altitudes is

$$q = ab^{\epsilon}$$

where

$q =$ the observed intensity of radiation,

$a =$ the intensity of radiation on unit surface outside the limits of the atmosphere,

$b =$ a "constant," which is the fraction showing the amount of absorption for a body in the zenith ; *i.e.*, the "absorption co-efficient,"

and

$\epsilon =$ the thickness of the atmosphere, the value being taken as unity for a body in the zenith. ϵ is approximately equal to sec. ZD. up to a zenith-distance of 60° or 65°.

In the case of the sun, a is the solar constant. One of the mistakes made by the older experimenters was that of assuming the quantity b to be really a constant, which it is not. It is, in fact, a function of two variables, viz., the wave-length of the radiation, and ϵ, the thickness of atmosphere traversed by the radiation. (LANGLEY, in commenting on this fact, seems to have overlooked ROSETTI's work, in which the increase of b with ϵ is clearly and quantitatively stated.)

From the results of his work, LANGLEY obtains 41 per cent. as a probable approximation to the absorption of total radiation for a body in the zenith. His argument may be briefly summarized thus:

The number of wave-lengths in a composite radiation is infinite. Each wave-length may have its own individual coefficient of absorption. The coefficients of absorption will be infinite in number and will vary in value between 0 and unity. As "some sort of adumbration of the complexity of nature's problem and the

* LANGLEY, 'Professional Papers of the Signal Service,' Washington, 1884, and 'Phil. Mag.,' 1884, vol. 18, p. 289.

method of his work," he divides the radiant energy before absorption into ten parts A, B, C, . . . J, each having its own coefficient of transmission, $a, b, c, \ldots j$, so that the total radiation outside our atmosphere being

$$A + B + C + D + \&c. \ldots = X,$$

the intensity after passing through unit thickness of air (*i.e.*, $\epsilon = 1$, a zenith observation) will be

$$Aa + Bb + Cc + Dd + \&c. \ldots = M,$$

after passing through two thicknesses ($\epsilon = 2$) will be

$$Aa^2 + Bb^2 + Cc^2 + Dd^2 + \&c. \ldots = N,$$

and so on, assuming that a, b, &c., remain constants for more than one integral value of ϵ, which is not exactly true.

Of course X is unknown from experiment, but M, N, O, &c., can be measured. Then the ratio N/M will give the transmission of the second thickness compared with the first, and 1 − N/M the absorption, and similarly with the other series, and these may all agree within close limits. The great mistake lay in assuming that if $\left(1 - \frac{N}{M}\right) = 1 - \left(\frac{O}{N}\right)^{\frac{1}{2}} = 1 - \left(\frac{P}{O}\right)^{\frac{1}{3}}$ *approximately*, then the same ratio held for the first thickness.

By giving values of $a, b, c \ldots$ &c. = ·01, ·1, ·2, ·6, ·7, ·7, ·8, ·9, ·9, and 1·0, while A = B = C = &c. = 1, LANGLEY shows that this equality of the ratios is at once destroyed, and holds that this rough division of the whole radiation into parts with varying coefficients of absorption, must give an approximation to the truth, Taking A = B = C = &c. = J = 1, the total outside radiation = 10, while

$$\begin{aligned} Aa + Bb + \ldots Jj &= 5{\cdot}9 = M \\ Aa^2 + Bb^2 + \ldots Jj^2 &= 4{\cdot}65 = N \\ Aa^3 + Bb^3 + \ldots Jj^3 &= 3{\cdot}88 = 0, \&c. \end{aligned}$$

Then

$$1 - \left(\frac{N}{M}\right) = {\cdot}21,\ 1 - \left(\frac{O}{N}\right)^{\frac{1}{2}} = {\cdot}19,\ 1 - \left(\frac{P}{O}\right)^{\frac{1}{3}} = {\cdot}18, \&c.,$$

while

$$1 - \frac{M}{X} = 1 - \frac{5{\cdot}9}{10{\cdot}0} = {\cdot}41,$$

so that instead of 21 per cent. being absorbed in one thickness of atmosphere, it may very well be *double* that absorption taking place.

We now come to an examination of ROSETTI's careful investigation on this point. He does not give the value of the absorption explicitly, but it may be deduced from the figures given by him on p. 546* of his paper already quoted.

* 'Phil. Mag.,' 1879.

From a large number of concordant observations he finally deduces a value of the solar constant = 323 in the scale divisions of his thermo-pile, while in the tables on p. 546 he gives the deflections corresponding to values of ϵ from 1·4 up to 4·8.

We plotted these values on curve paper (fig. 8), and thus found 229 as the corresponding deflection for the sun in the zenith, so that using the above symbols, X = 323, M = 229. The absorption for one thickness therefore equals

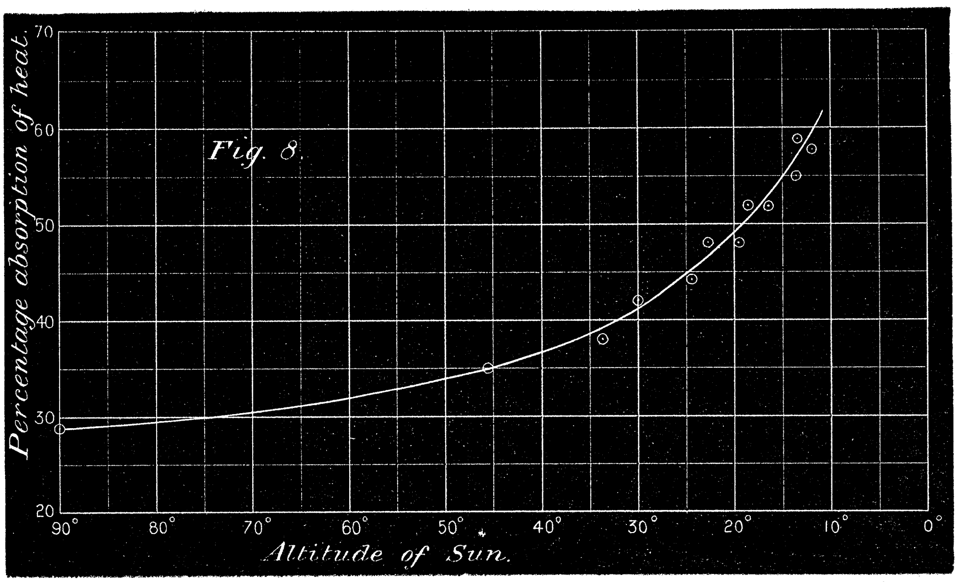
$$1 - \frac{M}{X} = 1 - \frac{229}{323} = 1 - \cdot 71 = \cdot 29.$$

So that 29 per cent. of the total outside radiation is absorbed, and 71 per cent. reaches the earth, with the sun in the zenith.

The ratios corresponding to other values of ϵ were similarly calculated, and the results plotted down, giving the curve (fig. 8), the abscissæ of which are zenith distances and the ordinates percentage absorptions.

The 29 per cent. thus deduced from Rosetti's results, it will be seen, is considerably greater than the old estimate, which we know to be incorrect, and less than the 41 per cent. of Langley, which is indeed a difference to be, *à priori*, expected for the following reason.

Fig. 8.



We know that by far the greater proportion of the energy (as properly measured by its heating effect) in the solar radiation is confined within narrow limits of wave-length, and that for these wave-lengths atmospheric absorption is less than for the waves of higher refrangibility. The larger transmission coefficients in Langley's calculations should therefore have more weight given to them, and it would be possible to draw up another series with assumed coefficients, by which the 29 per cent. could be reproduced, with the 21 per cent., 19 per cent., &c., following.

The difference then between ROSETTI'S and LANGLEY'S figures is in a direction which might be expected, and the results deduced from the work of the former may be assumed provisionally as an approximation to the truth.

Climatic conditions in Ireland are such as to entirely prevent a good series of observations on this point; a perfectly clear sky from morning to night, with a fairly constant hygrometic state of the atmosphere, is extremely rare.

ROSETTI, working under the unclouded skies of Northern Italy, was able to make a large number of observations at all hours of the day, with very consistent and apparently reliable results.

We have, therefore, determined to use the correcting factors for atmospheric absorption which have been deduced from his figures, so that whatever doubt may be thrown on the accuracy of his final result will affect ours in a certain proportion.

It is worth noting that YOUNG* gives 30 per cent. as the absorption in the zenith, but without indicating the means by which he arrives at this figure.

THE SOLAR RADIATION.

The general method of making the final experiments has already been described. The necessity for making observations with the sun shining (1) on the upper circuit of the radio-micrometer, (2) on the lower circuit, arises from the unavoidable difference in the constants of the two circuits. No special care had been taken in the construction of the instrument to make the receiving surfaces of equal size, and even if this had been possible, the electrical constants must have differed somewhat. The only way of correcting for these differences is to take independent observations in the manner indicated, and to take the *mean* of the results.

A considerable difference between the figures obtained in the two positions was to be anticipated, and it will be seen that experiment confirms the anticipation.

As we have already pointed out, when a balancing temperature has been obtained, the ratio of the radiation from the sun to that from the platinum is obtained by multiplying together four factors. They are:

(1) The ratio of the apparent area of the sun to that of the platinum, as seen from the receiving surface of the radio-micrometer. The former is obtained from the value of the sun's semi-diameter, as given by the 'Nautical Almanac' for the day of the observation. The latter is a constant, the same "stop" being always used in every position. The angle subtended by a diameter of the stop was $4^\circ{\cdot}702$;† if σ = angular diameter of the sun at the time of observation, we therefore have:—

$$\frac{\text{area of platinum}}{\text{area of sun}} = \left(\frac{4{\cdot}702}{\sigma}\right)^2.$$

* "The Sun," 'Internat. Sci. Series,' p. 262.

† A new stop was used after Sept. 8th; see p. 391.

(2) The ratio of the incident radiation on the glass mirror of the heliostat to the reflected. This was given by the use of FRESNEL's formula

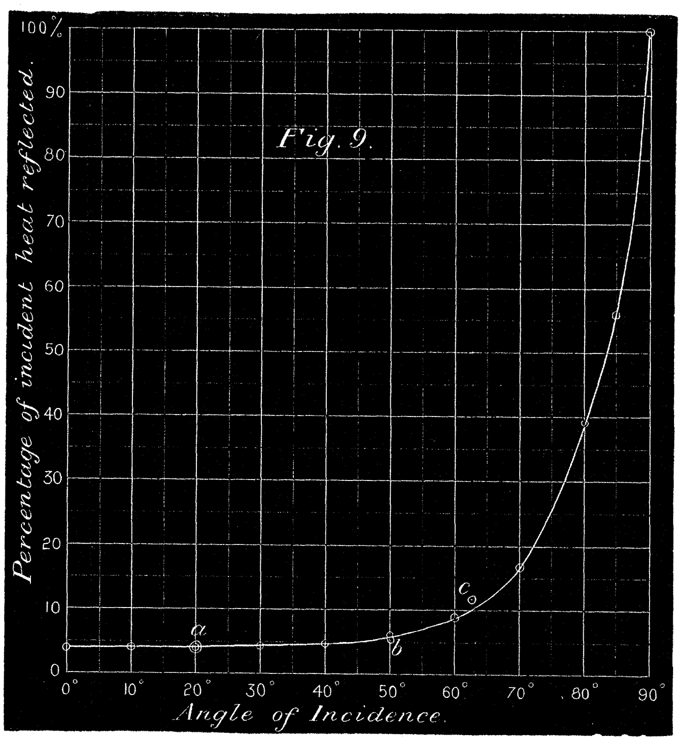
$$\frac{R_i}{R_r} = \frac{1}{2}\frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{1}{2}\frac{\tan^2(i-r)}{\tan^2(i+r)},$$

where

R_i = intensity of incident radiation,
R_r = ,, ,, reflected ,,
i = angle of incidence,
r = ,, ,, refraction, which was obtained by putting $\mu = 1\cdot5$ in the ordinary formula, $\sin i = \mu \sin r$.

The values thus obtained for different angles of incidence were plotted down and a smooth curve drawn to give the value at any incidence (fig. 9). In the figure, *a*, *b*,

Fig. 9.



and *c*, are points experimentally determined by photometric measurement as a rough check on the accuracy of the calculations. (It may be noted here that the table given by JAMIN* is erroneous as referring to common light; it is correct for light polarized in the plane of incidence. We mention this as anyone who took the accuracy of JAMIN's figures for granted would imagine that our curve was wrong.)

* 'Cours de Physique,' 4th edition, vol. 3, p. 618.

Sir J. CONROY* has shown that the curve drawn from FRESNEL'S formula is verified by experiment to within $\frac{1}{2}$ per cent. at the angles of incidence generally used in our observations.

The angle of incidence is obtained at each experiment by finding the distance between the end of a certain steel rod in the heliostat and a collar which slides along it; the angle corresponding to any distance could be found by means of a curve, which it is unnecessary to give here.

(3) The ratio of the radiation outside our atmosphere to the amount which reaches the earth. This is obtained by calculating the altitude from the known declination, hour angle, and latitude, and taking the percentage of absorption from the curve (fig. 8) which we have already discussed.

(4) The ratio of the emissivity of bright platinum to that of a lamp-blacked surface, which, as already mentioned, we take as 35 : 100.

To take a typical case :—

Date, Sept. 4th, 1893. ☉ Declination = 7°·1 N. ☉ $\frac{1}{2}$ diameter = 15′·9.
Time, 10^{h} 54^{m}, local. Therefore ☉ altitude = 41°·8.
Balancing temperature = 1514° absolute.
By curve (fig. 8) absorption = 36 per cent.
Therefore transmission = 64 per cent.
Diameter of ☉ = 31′·8 = 0°·53.
Therefore

$$\frac{\text{Area of platinum}}{\text{Area of sun}} = \left(\frac{4\cdot702}{\cdot53}\right)^2 = 78\cdot71.$$

Angle of incidence on glass = 61°.
Therefore amount of heat reflected = 9·5 per cent.
Ratio of emissivity of platinum and lamp black = $\frac{35}{100}$.
Therefore the radiation from the sun is

$$78\cdot71 \times \frac{100}{64} \times \frac{100}{9\cdot5} \times \frac{35}{100} = 453\cdot1$$

that of the platinum at a temperature of 1514° absolute.

The temperature of the sun is therefore

$$1514 \times \sqrt[4]{453\cdot1} = 1514 \times 4\cdot614 = 6985° \text{ absolute,}$$

according to this single observation.

It was not only necessary to take observations with the sun shining (A) into the lower circuit and (B) into the upper circuit, but, on account of possible differences in the state of the surfaces, back and front, of the copper foil receivers, it was essential

* 'Phil. Trans.,' 1889, (A), vol. 180, p. 245.

to turn the whole radio-micrometer through an angle of 180°, so that the heat from the platinum should now fall on that side of the receiving surfaces on which previously the sun had shone. The different positions are distinguished as follows:—

Position (1A) = platinum heating upper circuit, and behind the small mirror fixed to the fibre of the radio-micrometer.
,, (1B) = platinum heating lower circuit, and again behind mirror.
,, (2A) = instrument rotated through 180°; platinum in upper circuit, *in front* of mirror.
,, (2B) = platinum in lower circuit, again in front of mirror.

The difference between positions 1A and 2A, and between 1B and 2B, we should, *à priori*, expect to be small, and the experiments show that this is so, while, as we have already mentioned, the larger differences between the A and B positions were also to be anticipated from unavoidable dissimilarities in the two parts of the combined circuit.

One further point remains to be noticed, viz., that the geometrical mean of the mean temperatures of the A and B positions is not exactly the mean temperature to be deduced from the observations, on account of the curvature of the radiation curve.

To show what difference exists between the geometrical and the true mean, we may take the following numerical example:

Mean balancing temperature in position A = 1600° absolute
,, ,, ,, B = 1300° ,,

Mean balancing temperature = 1443°

Now, to a temperature of 1600°, corresponds a radiation of 312 in our arbitrary units; to a temperature of 1300°, a radiation of 136; mean radiation $= \frac{1}{2}(312 + 136) = 224$. But to a radiation of 224, corresponds a temperature of 1472°, which is 29° higher than the geometrical mean, and the value of $\sqrt[4]{\left(\frac{\text{Radiation of sun}}{\text{Radiation of platinum}}\right)} = 4{\cdot}5$ approximately. That is to say, we must add $29 \times 4{\cdot}5 = 130$ to the mean temperature. A correction of about 100° is therefore to be made on the final mean of all the observations, the separate details of which now follow. Each day's results are given by themselves, with data sufficiently full to allow of any single observation being calculated out.

The date, height of barometer, and notes on the weather are given first; hygrometrical readings are not given, as no useful deductions can be made from them, as ROSETTI points out in his paper.

In the 1st column, the position is noted.
,, 2nd ,, the local time of the observation.

In the 3rd column, the readings on the meldometer scale at the moment of balance.
,, 4th ,, the absolute temperature corresponding to this reading.
,, 5th ,, the sun's altitude.
,, 6th ,, the percentage of transmission of the total solar heat through the earth's atmosphere.
,, 7th ,, the angle of incidence of the sunlight on the mirror of the heliostat.
,, 8th ,, the percentage reflection of the heat in the incident beam.
,, 9th ,, the absolute temperature of the sun as calculated from each single observation.

Date: September 3rd, 1893.
Weather: Passing clouds. Sky, no perceptible haze. Barometer, 30·2 in.

Position.	Local time.		Balance.		Sun's altitude.	Per cent. trans.	Angle of incidence.	Per cent. reflected.	Sun's abs. temp.
			Reading.	Temp. abs.					
	h.	m.		°	°		°		°
1A	10	3	72·4	1474	38·0	62·5	57·0	7·6	7176
	10	6	70·5	1447					7044
	10	21	74·3	1513	39·6	63·0	58·8	8·0	7367
	10	23	73·8	1503					7318
	10	40	76·1	1543	41·3	63·8	60·0	8·9	7242
	10	43	76·2	1544					7247
	10	44	76·5	1547					7261
								Mean .	7236
1B	10	29	56·6	1223	40·2	63·5	59·4	8·5	5813
	10	30	56·2	1214					5770
	10	32	56·6	1223					5813
	10	33	56·4	1218					5789
	10	51	58·2	1246	42·1	64·0	60·5	8·9	5826
	10	52	58·6	1250					5845
	10	53	58·5	1249					5841
	0	52	65·0	1361	43·6	64·5	66·5	13·2	5771
	0	54	64·2	1346					5708
								Mean .	5797

NOTE.—New platinum strip put in after these observations were made. Balance readings here refer to Calibration-line 2.

Date : September 4th, 1893.

Weather : Hazy clouds, with intervals of light blue sky ; Wind S.S.E., moderate.

Position.	Local time.	Balance.		Sun's altitude.	Per cent. trans.	Angle of incidence.	Per cent. reflected.	Sun's abs. temp.
		Reading.	Temp. abs.					
	h. m.		°	°		°		°
1A	10 54	67·6	1514	41·8	64·0	61·0	9·4	7003
	10 56	67·2	1508	41·8	64·0	61·0	9·4	6975
	10 57	69·0	1539	41·8	64·0	61·0	9·4	7119
	11 35	71·2	1581	43·0	64·3	63·6	11·0	7022
	11 36	71·2	1581	43·0	64·3	63·6	11·0	7022
	11 37	72·0	1594	43·0	64·3	63·6	11·0	7053
							Mean .	7032
1B	11 10	53·2	1254	42·5	64·0	62·2	10·1	5697
	11 27	53·3	1256	42·5	64·0	62·2	10·1	5706
	11 28	53·3	1256	42 5	64·0	62·2	10·1	5706
	11 43	54·0	1268	43·2	64·3	63·8	11·0	5632
	11 51	54·0	1268	43·2	64·3	63·8	11·0	5632
	11 53	54·2	1273	43·2	64·3	63·8	11·0	5654
							Mean .	5671

NOTE.—Balance readings refer to Calibration-line 3.

Date : September 7th, 1893.

Weather : Passing clouds, with intervals of clear blue sky ; Wind W., moderate. Barometer, 29·7 in.

Position.	Local time.	Balance.		Sun's altitude.	Per cent. trans.	Angle of incidence.	Per cent. reflected.	Sun's abs. temp.
		Reading	Temp. abs.					
	h. m.		°	°		°		°
1A	10 54	67·9	1519	40·6	63·6	61·6	10·0	6930
	10 56	68·2	1525	40·6	63·6	61·6	10·0	6957
	10 57	68·5	1531	40·6	63·6	61·6	10·0	6984
							Mean .	6957
1B	11 15	53·7	1263	41·4	64·0	62·6	10·5	5683
	11 17	54·2	1273	41·4	64·0	62·6	10·5	5728
	11 18	54·2	1273	41·4	64·0	62·6	10·5	5728
							Mean .	5713
2A	11 53	75·8	1663	42·2	64·2	65·4	12·4	7173
	11 54	78·0	1703	42·2	64·2	65·4	12·4	7345
	0 5	80·0	1741	42·2	64·2	66·0	13·0	7370
	1 15	81·2	1762	39·6	63·0	67·5	14·2	7381
	1 26	82·2	1778	39·6	63·0	67·5	14·2	7448
							Mean .	7343
2B	0 42	62·5	1423	41·4	64·0	67·0	14·0	5959
	0 44	62·3	1420	41·4	64·0	67·0	14·0	5946
	0 45	63·0	1421	41·4	64·0	67·0	14·0	5992
	1 58	59·5	1368	36·3	61·9	66·6	13·6	5818
	2 0	59·7	1371	36·3	61·9	66·6	13·6	5831
	2 1	59·0	1359	36·3	61·9	66·6	13·6	5780
							Mean .	5884

Date: September 8th, 1893.

Weather: Generally so cloudy that very few observations were possible. Barometer, 29·5 in.

Position.	Local time.	Balance.		Sun's altitude.	Per cent. trans.	Angle of incidence.	Per cent. reflected.	Sun's abs. temp.
		Reading.	Temp. abs.					
	h. m.		°	°		°		°
2B	11 44	60·0	1378	41·8	63·9	65·0	12·0	5982
	11 45	60·0	1378	41·8	63·9	65·0	12·0	5982
	11 46	60·5	1386	41·8	63·9	65·0	12·0	6016
							Mean .	5993

NOTE.—After the above observations had been made, the aperture through which the radiations from the platinum passed into the radio-micrometer was enlarged, as in some cases the balancing temperature became inconveniently high. The dimensions of the new aperture were:—

Diameter = 5·57 millims. Angle subtended = 5°·301.

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Date: September 10, 1893.

Weather: Cold N.E. wind, with very slight haze. Barometer, 29·9 in.

Position.	Local time.	Balance.		Sun's altitude.	Per cent. trans.	Angle of incidence.	Per cent. reflected.	Sun's abs. temp.
		Reading.	Temp. abs.					
	h. m.		°	°		°		°
2A	0 8	71·2	1578	41·1	63·9	66·3	13·3	7096
	0 12	71·5	1583	41·1	63·9	66·3	13·3	7119
	0 13	71·6	1585	41·1	63·9	66·3	13·3	7127
	0 33	71·6	1585	40·0	63·4	67·2	14·1	7038
	0 47	71·8	1590	40·0	63·4	67·2	14·1	7060
	0 48	71·5	1583	40·0	63·4	67·2	14·1	7029
	1 5	75·0	1646	39·3	63·0	68·0	15·0	7208
	1 7	74·0	1629	39·3	63·0	68·0	15·0	7133
	1 10	73·0	1612	39·3	63·0	68·0	15·0	7058
	1 15	74·0	1629	39·3	63·0	68·0	15·0	7133
							Mean .	7100
2B	0 20	53·2	1254	41·0	63·8	66·5	13·5	5620
	0 21	52·6	1243	41·0	63·8	66·5	13·5	5571
	0 22	52·8	1245	41·0	63·8	66·5	13·5	5580
	0 55	54·7	1283	39·5	63·1	67·2	14·1	5704
	0 58	55·8	1303	39·5	63·1	67·2	14·1	5792
	0 59	55·0	1289	39·5	63·1	67·2	14·1	5731
	1 22	54·4	1276	38·4	62·7	67·6	14·5	5642
	1 23	54·2	1273	38·4	62·7	67·6	14·5	5629
							Mean .	5659
1B	1 46	55·5	1297	36·2	61·9	67·6	14·5	5754
	1 48	55·0	1289	36·2	61·9	67·6	14·5	5719
	1 49	55·0	1289	36·2	61·9	67·6	14·5	5719
							Mean .	5731
1A	1 53	71·2	1578	35·5	61·4	67·6	14·5	7014
	1 54	71·0	1576	35·5	61·4	67·6	14·5	7005
	1 56	71·0	1576	35·5	61·4	67·6	14·5	7005
							Mean .	7008

MEANS OF DAILY MEANS.

Position 1A . . .	7236° . . .	7 observations.
	7032° . . .	6 ,,
	6957° . . .	3 ,,
	7008° . . .	3 ,,
Mean . . .	7058°	
Position 2A . . .	7343° . . .	5 observations.
	7100° . . .	10 ,,
Mean . . .	7222°	

Mean of 1A and 2A = 7140°.

Position 1B . . .	5797° . . .	9 observations.
	5671° . . .	6 ,,
	5713° . . .	3 ,,
Mean . . .	5727°	
Position 2B . . .	5884° . . .	9 observations.
	5993° . . .	6 ,,
	5659° . . .	3 ,,
Mean . . .	5639°	

Mean of 1B and 2B = 5683.
,, 1A and 2A = 7140.

Therefore

$$\text{Mean result} = \sqrt{5683 \times 7140} = 6370^\circ \text{ absolute.}$$

To this 100° must be added for the reason on page 387.

As there must necessarily be errors of observation, and as results on different days give values differing by as much as 300°, chiefly owing, no doubt, to a change in atmospheric conditions, it has been considered unnecessary to go into certain refinements in the calculation such as using the method of least squares. The daily means have also been given equal weights, in spite of differences in the number of observations.

The geometrical instead of the arithmetical mean of the calculated temperature in the A and B positions, is taken for the following reason :

Let

R_s = radiation due to sun falling on *unit area of receiving surface ;*

R_{p^1} and R_{p^2} = respective radiations due to platinum, also on unit area, when giving heat—(1) to the upper surface ; (2) to the lower. R_s will, of course, be the same in the two positions ;

a_1 = effective area of upper surface ;

a_2 = ,, ,, lower ,,

using the word "effective" to cover any slight difference of absorptive power, &c.

Then, if we suppose, *First*, the radiation due to the sun falling on the upper surface, the lower being sheltered from the platinum, we should have a deflection θ_1, and as deflections may be taken proportional to received radiation, then

$$a_1 R_s = m\theta_1$$

where m is a constant.

Secondly, let the radiation from the platinum fall on the lower circuit, the sun being now cut off from the upper ; we shall have

$$a_2 R_{p_2} = m\theta_2.$$

But if both effects are allowed to be produced together, at the moment of balance θ_1 and θ_2 will be equal and opposite, and therefore

$$a_1 R_s = a_2 R_{p_2}.$$

Similarly, with the sun and platinum reversed as regards the upper and lower surfaces, while R_s remains the same, R_p becomes R_{p_1}, and we have

$$a_2 R_s = a_1 R_{p_1},$$

which gives immediately

$$R_s = \frac{R_{p_1} R_{p_2}}{R_s},$$

or

$$R_s = \sqrt{R_{p_1} R_{p_2}},$$

from which the reason for taking the geometrical mean of the corresponding temperatures follows directly.

The final result, therefore, arrived at, is only given to the nearest 100 ; it is

6200° C.

In conclusion, we may point out that this method would probably give excellent results, if a series of observations were undertaken to settle the question of how, or if, the solar temperature varies during a sun-spot cycle. The instrument should, of course, be used in or near the tropics, where atmospheric conditions can be trusted to remain more constant than in this country. Any error in the absolute value obtained might probably be considered constant, so that comparative values from year to year might be trusted to indicate any change.

NOTE, ADDED APRIL 13TH, 1894.

It has been mentioned in the paper that ROSETTI's determination of the amount of the (terrestrial) atmospheric absorption has been used in the calculations of the effective solar temperature. It may be well, however, to give the result obtained by using other estimates of this quantity, which (after the law connecting radiation and temperature) is the most important factor in the final value.

Taking LANGLEY's estimate for zenith absorption, 41 per cent., instead of ROSETTI's, 29 per cent., the respective transmission coefficients being therefore 59 per cent. and 71 per cent., the temperature would be multiplied by $\sqrt[4]{(71/59)}$ approximately; *i.e.*, instead of 6200°, we should obtain

$$6200 \times \sqrt[4]{(71/59)} = 6200 \times 1{\cdot}054 = 6535^\circ \text{ C.}$$

But a later, and still higher, estimation of the zenith absorption has been made. ANGSTRÖM ('WIED. Ann.,' 1890, vol. xxxix., p. 309) has shown that the effect of the carbonic acid gas in the atmosphere is much more important than had hitherto been supposed, and obtains 64 per cent., as against ROSETTI's 30 per cent. and LANGLEY's 41 per cent. This gives 36 per cent. as the transmission coefficient, and, taking this value, the temperature becomes*

$$6200 \times \sqrt[4]{(71/36)} = 6200 \times \sqrt[4]{(2)} \text{ approximately} = 6200 \times 1{\cdot}189 = 7370^\circ.$$

And, to make the case general, if any later investigation shows the zenith transmission coefficient to be X per cent., the effective temperature becomes

$$6200 \times \sqrt[4]{(71/X)}.$$

It may also be of interest to see what effect is produced if absorption in the atmosphere of the sun itself is taken into account. First, considering the falling-off in radiation from the central to the peripheral parts of the sun's disc, from WILSON and RAMBAUT's paper "On the Absorption of Heat in the Sun's Atmosphere" ('Proc. R.I.A.,' 1892, 3rd series, vol. 2, p. 299), we may deduce that, if the absorption were

* The ratio of the zenith-absorptions is practically equal to that of those with a greater thickness of atmosphere, at least down to a zenith-distance of 50°.

everywhere equal to that at the centre, the radiation would be increased by 4/3, and the temperature would become approximately

$$7370 \times \sqrt[4]{(4/3)} = 7370 \times 1{\cdot}074 = 7900^\circ.$$

Secondly, assuming WILSON and RAMBAUT's result for the *total* loss due to absorption in the solar atmosphere—viz., that about one-third of the radiation is cut off—the radiation would be multiplied by 3/2 if the sun's atmosphere were removed, and our estimate of the temperature would have to be multiplied by $\sqrt[4]{(3/2)}$, so that (again taking the highest value given above as being probably nearest the truth) we get finally

$$7900 \times \sqrt[4]{(3/2)} = 7900 \times 1{\cdot}107 = 8740^\circ.$$

We may therefore summarize as follows:—

Effective temperature of the sun, taking

(1) ROSETTI's estimate of loss in the earth's atmosphere . = 6200° C.
(2) LANGLEY's estimate . . . . = 6500° C.
(3) ÅNGSTRÖM's estimate . . . . = 7400° C.

And finally, considering the probable effect of the sun's own atmosphere, allowing for it by the figures given in WILSON and RAMBAUT's paper already quoted, and using the highest value just obtained, the effective temperature comes out as approximately 8700° C.

NOTE, ADDED JULY 24TH, 1894.

Some investigations by the authors in connection with the temperature of the carbon of the electric arc, which are now in progress, lead to the conclusion that the simple fourth-power law of radiation used above is only an approximation to the truth, closer in the case of bare platinum than in that of blackened, so that the assumption made in the paper that both follow the same law is not strictly correct. The new work will shortly be published, and will probably result in raising by a few hundred degrees the value obtained above. It may be noticed, meanwhile, that the experimental figures given in this paper are sufficient to serve as a basis—whatever law of radiation may be used—from which the solar temperature may be calculated with an accuracy increasing with a growth of more accurate knowledge as to the law of radiation, and the amount of the atmospheric absorption.